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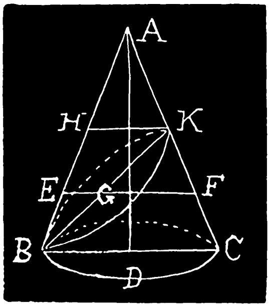
CALCULUS.

185. Proposed by A. H. HOLMES, Brunswick, Maine.

Required the perpendicular height of a right cone, radius of base being unity, such that the maximum ellipse that can be cut from the cone shall equal the base of the cone in area.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.



Let $BC=2=c$; a, b the semi-axes of the ellipse, $\angle KBC=\theta$, $\angle DAC=\beta$. Then $BK=2a=c\cos\beta/\cos(\theta-\beta)$ or $a=\frac{1}{2}c\cos\beta/\cos(\theta-\beta$ $b^2=EG.GF=\frac{1}{4}cHK$. But $HK=2a\cos(\theta+\beta)/\cos(\theta-\beta)$.

$\therefore b=\frac{1}{2}c\sqrt{[\cos(\theta+\beta)/\cos(\theta-\beta)]}$, and therefore $\pi ab=\frac{1}{4}\pi c^2\cos\beta\sqrt{[\cos(\theta+\beta)]}/[\cos(\theta-\beta)]^{\frac{3}{2}}=\text{maximum}$.

$\therefore \dfrac{\sqrt{[\cos(\theta+\beta)]}}{\cos(\theta-\beta)\sqrt{[\cos(\theta-\beta)]}}=\text{maximum}$, and $\sin2\theta=2\sin2\beta$, or, $\sin\theta=\frac{1}{2}\sqrt{\{2[1+\sqrt{(1-4\sin^2 2\beta)}]\}}$.

Since $c=2$ and $\pi ab=\pi c^2/4$, we have $\cos^2\beta\cos(\theta+\beta)=\cos^3(\theta-\beta)$. Now $\cos\theta=\frac{1}{2}\sqrt{\{2[1-\sqrt{(1-4\sin^2 2\beta)}]\}}$,

$$\cos^3\beta\sin\theta\cos\theta=4\sin\beta\cos^2\beta\cos^2\theta+3\sin^2\beta\cos\beta\sin\theta\cos\theta+\sin\beta\sin^2\theta.$$

Substituting the values of $\sin\theta$, $\cos\theta$ and reducing,

$$4\sin^2 2b+1=(2\cos2\beta+1)\sqrt{(1-4\sin^2 2\beta)}.$$

$\therefore 16\sin^4 2\beta+40\sin^2 2\beta=7$, and $\sin2\beta=\frac{1}{2}\sqrt{(4\sqrt{2}-5)}$; $\beta=11^\circ\ 57'\ 9''$.
$\therefore$ Altitude$=\cot\beta=4.72388$.

Also solved by J. Scheffer, the Proposer, and L. E. Newcomb.

186. Proposed by Editor EPSTEEN.

Evaluate $\displaystyle\int_0^\infty \frac{\sin my}{y}dy$, $\displaystyle\int_0^\infty \frac{\cos my}{y}dy$.

I. Solution by IDA M. SCHOTTENFELS, M. A., New York City.

$\displaystyle\int_0^\infty \frac{\sin my}{y}dy=+\frac{1}{2}\pi,\ 0,\ -\frac{1}{2}\pi$, according as $m>0$, $m=0$, $m<0$ if $y>0$.

Proof. If $y>0$ then $\displaystyle\int_0^\infty e^{-yx}dx=1/y$. Multiplying both members by $\sin my$:

$$(1)\quad \sin my\int_0^\infty e^{-yx}dx=\frac{\sin my}{y};\qquad (2)\quad \int_0^\infty e^{-yx}\sin my\,dx=\frac{\sin my}{y}.$$

Integrating (2) with respect to y between 0 and ∞,

$$(3)\qquad \int_0^\infty \left(\int_0^\infty e^{-yx}\sin my\, dx\right)dy = \int_0^\infty \frac{\sin my}{y}dy.$$

Changing the order of integration since the limits are the same we have

$$\int_0^\infty \left(\int_0^\infty e^{-yx}\sin my\, dy\right)dx = \int_0^\infty \frac{\sin my}{y}dy.$$

But

$$(5)\qquad \int_0^\infty e^{-yx}\sin my\, dy = \frac{m}{m^2+x^2}\quad (y>0).$$

The result in (5) is obtained by finding the indefinite integral and passing to the limit thus,

$$\int e^{-yx}\sin my\, dy = -e^{-yx}\left[\frac{m\cos my + x\sin my}{x^2+m^2}\right]$$

$$\lim_{y \doteq \infty} -e^{-yx}\left[\frac{m\cos my + x\sin my}{x^2+m^2}\right] = \frac{m}{x^2+m^2}\quad (y>0)$$

$$(6)\qquad \int_0^\infty \frac{m}{x^2+m^2}dy = \int_0^\infty \frac{\sin my}{y}dy.$$

$$(7)\qquad \int_0^\infty \frac{m}{x^2+m^2}dx = \tan^{-1}\frac{x}{m}\Big]_0^\infty.$$

$\tan^{-1}\frac{x}{m}\Big]_0^\infty = \frac{1}{2}\pi,\ 0,\ -\frac{1}{2}\pi$ according as $m>0$, $m=0$, $m<0$, and therefore $\int_0^\infty \frac{\sin mx}{x}dx$ has the same values and equation (1) is true.

We now regard the given integral as a function of the parameter m thus

$$\int_0^\infty \frac{\sin my}{y}dy = \varphi(m)$$

and trace the resulting curve.

For all negative values of m the integral $\int_0^\infty \frac{\sin mx}{x}dx$ describes a line parallel to the m axis and at a distance $-\frac{1}{2}\pi$ from it until m reaches the value 0; here the function has a finite discontinuity and jumps from $-\frac{1}{2}\pi$ to $+\frac{1}{2}\pi$, and for positive values of m continues in a straight line parallel to the m axis and at a distance $+\frac{1}{2}\pi$ from it.

In a similar manner $\int_0^\infty \frac{\cos my}{y}dy = \infty$ for real finite values of m.

II. Solution by S. A. COREY, Hiteman, Iowa.

We have $1/y=\int_0^\infty e^{-ay}da$ if $y>0$. Hence

$$\int_0^\infty \frac{\sin my}{y}dy=\int_0^\infty \sin my\int_0^\infty e^{-ay}da\ dy=\int_0^\infty\int_0^\infty \sin my\ e^{-ay}\ da\ dy,$$

$$=\int_0^\infty\int_0^\infty \sin my\ e^{-ay}\ dy\ da=\int_0^\infty \frac{m da}{a^2+m^2}$$

$$=\tfrac{1}{2}\pi \text{ if } m>0,\ =-\tfrac{1}{2}\pi \text{ if } m<0,\ =0 \text{ if } m=0.$$

(See Byerly's *Integral Calculus*, page 100.) Similarly,

$$\int_0^\infty \frac{\cos my}{y}dy=\int_0^\infty \cos my\int_0^\infty e^{-ay}\ da\ dy=\int_0^\infty \frac{a da}{a^2+m^2}=0,$$

for all finite real values of m.

III. Remark by F. D. POSEY, A. B., San Mateo, Cal.

A discussion of the first of these integrals will be found on page 271, Art. 285 of Todhunter's *Integral Calculus*. The result there obtained is $\frac{1}{2}\pi$ if m be positive, $-\frac{1}{2}\pi$ if m be negative.

Also solved by G. W. Greenwood, G. B. M. Zerr, and J. Scheffer.

MISCELLANEOUS.

146. Proposed by F. P. MATZ, Ph. D., Sc. D.

Given $\left\{\begin{matrix} a\cos\alpha+b\sin\alpha=c \\ a\cos\beta+b\sin\beta=c\end{matrix}\right\}$ to prove that

$$\sin(\alpha+\beta)=\frac{2ab}{a^2+b^2},\text{ and } \cot\alpha+\cot\beta=\frac{2ab}{c^2-a^2}.$$

I. Solution by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.

We have at once $\dfrac{\cos\alpha-\cos\beta}{\sin\alpha-\sin\beta}=-b/a$ ....(1).

Now $\cos\alpha-\cos\beta=-2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\alpha-\beta)$ and $\sin\alpha-\sin\beta=2\cos\frac{1}{2}(\alpha+\beta)\times\sin\frac{1}{2}(\alpha-\beta)$. Substituting these values in (1) we obtain $\tan\frac{1}{2}(\alpha+\beta)=b/a$. Since $\tan\frac{1}{2}(\alpha+\beta)=\pm\sqrt{\dfrac{1-\cos(\alpha+\beta)}{1+\cos(\alpha+\beta)}}=b/a$, and solving,

$$\cos(\alpha+\beta)=\frac{a^2-b^2}{a^2+b^2}\text{.........(2)};\ \sin(\alpha+\beta)=\frac{2ab}{a^2+b^2}\text{.........(3)}.$$

Multiplying together the given equation and substituting the values of $\cos(\alpha+\beta)$,